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SETTLING VELOCITIES OF SIMPLE BEAD AGGLOMERATES AT LOW REYNOLDS NUMBERS

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Abstract: Theoretical and experimental investigations on settling velocities of bead agglomerates in viscous regime were accomplished. Agglomerates were constructed from 6-17 spherical beads in form of flocs in various configurations. Theoretical analysis was preformed using computer code WinHYDRO++. Settling velocities were correlated with fractal dimensions and gyration radii of the agglomerates. Winterwerp's formula, based on Stokes equation, for settling velocity of cohesive agglomerates was verified. Laboratory measurements of settling velocities were done in sedimentation column of height 95.0 cm and diameter 9.0 cm filled with glycerin. Settling velocities calculated using WinHYDRO++ have overestimated, on average by 24%, the measured values and those calculated by the Winterwerp's formula have underestimated the measured ones by 22%. Radius of gyration has occurred a more adequate parameter to describe the settling velocity than fractal dimension.
KEY WORDS: bead agglomerates, fractal dimension, settling velocity, WinHYDRO++

NOTATION

d_a – aggregate diameter, m
d_p – diameter of elementary bead (primary particle), m
D_{nf} – fractal dimension, -
g – acceleration due to gravity, m/s^2
k_ψ – sphericity, -
m – mass of the whole agglomerate, g
N – number of primary beads in an agglomerate, -
r_i – distance of the i-th primary particle from the axis of gyration, m
V_1 – settling velocity of a single particle, m/s
μ_l – dynamic viscosity of liquid, Pa·s
ν – kinematic viscosity, m^2/s
$\rho_{s(l)}$ – density of floc material (liquid), kg/m^3

1. INTRODUCTION

Settling velocity of agglomerates is a basic parameter in description of sedimentation in both natural and artificial conditions. Gravitational settling is widely acknowledged as a key mechanism behind removal of suspended particles from the water column in aquatic environments such as stormwater and fish ponds, lakes, river deltas, estuaries and marine environments as well as water and wastewater treatment installations (Winterwerp 1998, Khelifa 2006). Small sediment particles aggregate to form large flocs which sink much faster in water than the primary particles.

Attempts to model settling velocity as a function of floc size, shape and density have been undertaken for more than hundred years (Smoluchowski 1911). These early attempts to predict the settling velocity of natural flocs were often erroneous ones and demonstrated that the assumption of size-invariant density was incorrect, which forced researchers to modify their formulae using empirical factors.

Due to irregular shapes of some agglomerates it is hard to calculate their settling velocities (also during centrifugation), which are commonly employed as a source of information about the overall structure and dynamic behavior of biological and synthetic macromolecules in solution (Banachowicz 2013). Biological macromolecules, cohesive agglomerates and other similar flocs, usually exhibit complex shapes that can not be represented by simple geometries, like ellipsoids or cylinders, for which hydrodynamic properties can be calculated from analytical formulae.

One of relatively new methods, developed under conditions of viscous settling of rigid macromolecules or particles of arbitrary shapes, is so called bead modelling. A particle (agglomerate) is represented here as an assembly of primary spherical elements (Bloomfield et al. 1967, Garcia de la Torre et al. 1978). In this work a system of modified hydrodynamic interaction equations was used to calculate settling velocity of several chosen particles made of equal primary beads.

Winterwerp's formula, based on Stokes equation, for settling velocity of cohesive agglomerates, in the following form (Winterwerp 1998):

$$V_W = g\, k_\psi \,\frac{\rho_s - \rho_l}{18\mu_l}\, d_p^{3-Dnf}\, \frac{d_a^{\,Dnf-1}}{1+0.15\mathrm{Re}_{d1}^{0.687}} \tag{1}$$

where: $\mathrm{Re}_{d1} = \rho_l V_W d_a \mu_l^{-1}$, was also verified. Winterwerp did not determine exactly the range of validity of his formula, however from data in his paper (Winterwerp 1998) it can be deduced that it holds for $1.4 < D_{nf} < 2.3$.

Both calculation methods were verified on the basis of our own experiments and some data from the literature.

2. MATERIALS AND METHODS

Physical models of agglomerates were made of plastic glued beads and beads connected with thin steel rods. Different particle models were created from 6 and 13 primary beads (Fig. 1); their characteristics are shown in Tab. 1.

Fig. 1. Views of physical bead models: a) 6-bead model (linear), b) 6-bead model (spatial), c) 13-bead model,

Table 1

Characteristics of physical model particles

Model particle	Mass	Volume	Mean density ρ_s	Diameter of particle	
				primary d_p	model d_a
	g	cm^3	g/cm^3	cm	cm
6-bead (I)	0.7678	0.6738	1.140	0.595	3.60
1-bead (I)	0.1285	0.1123	1.144	0.595	-
6-bead (II)	0.7630	0.6761	1.129	0.595	3.70
1-bead (II)	0.1281	0.1139	1.125	0.595	-
6-bead (linear)	0.6816	0.6614	1.031	0.595	3.57
13-bead	1.4768	1.4330	1.031	0.595	2.97

Measurements of translational velocity of the above described particles under gravity were made in a cylinder of diameter 9.0 cm and height 95.0 cm filled with glycerin of density 1.262 g/cm^3 at temperature 20°C. Due to the higher density of the liquid than density of the particles, a model particle was placed at the bottom of the cylinder using a special looped wire. Therefore, instead of settling, a flotation time was measured over vertical distance equal to 45 cm. Every measurement was repeated several times.

Fractal dimension was estimated using the following relationship (Błażejewski 2015):

$$D_{nf} \approx \frac{\log N}{\log \dfrac{d_a}{d_p}} \tag{2}$$

Calculations of translational velocity of the model bead particles (agglomerates) were made using a public-domain computer code WinHYDRO++ developed by Garcia de la Torre et al. (1997, 2010) on the base of a theory elaborated by Bloomfield at al. (1967). Model particles were created in the code and their settling velocities were calculated under conditions of viscous regime. The mass of agglomerates was expressed in daltons. Dynamic viscosity of glycerin at temperature 20^{o}C was taken as 14.10 P, i.e. 1.4 Pas. Other data and results of calculations are shown in Table 2. Sedimentation coefficient, expressed in svedbergs, was recalculated to the translational velocity under gravitational acceleration.

To compare results obtained with the use of WinHYDRO++, further calculations by the Winterwerp's formula (1998) were accomplished. Due to implicit form of the formula, an iterative procedure was applied. Sphericity coefficient k_ψ = 1 was utilized, as all the particles were built from spherical primary particles. The Reynolds number, based on the measured floating velocity V_e, was expressed as:

$$\mathrm{Re} = \frac{V_e \cdot d_a \cdot \rho_l}{\eta} = \frac{V_e \cdot d_a}{\nu} \tag{3}$$

3. RESULTS AND DISCUSSION

Particle characteristics, and translational velocities obtained from physical experiments, as well as calculated by WinHYDRO++ and Winterwerp's formula are presented in Table 2.

Table 2

Particle characteristics and their translational velocities under gravity

Model particle	Mass		Fractal dimension D_{nf}	Measured mean velocity V_e	Velocity by WinHYDRO++ V_{WH}	Velocity by Winterwerp V_W	Re
	g	Da	-	mm/s	mm/s	mm/s	-
6-bead(I)	0.7678	4.625E+23	1.00	2.06±0.02	2.62	1.65	5.008E-02
1-bead (I)	0.1285	7.741E+22	-	1.66±0.02	1.31	-	1.049E-02
6-bead (II)	0.7630	4.60E+23	0.98	1.72±0.02	2.76	1.63	5.068E-02
1-bead (II)	0.1281	7.71E+22	-	1.33±0.04	-	-	8.406E-03
6-bead linear	0.6816	4.11E+23	1.00	6.30±0.31	6.36	3.08	9.270E-02
1-bead*	0.1136	0.69E+23	-	-	2.67	-	5.029E-02
13-bead	1.4768	8.89E+23	1.59	8.30±0.06	9.09	7.41	1.857E-01
9-bead	1.0224	6.157E+23	1.37	-	8.55	5.17	1.296E-01
15-bead	1.7040	1.026E+24	1.39	-	10.27	6.36	2.229E-01
17-bead	1.9312	1.163E+24	1.46	-	10.30	6.85	2.402E-01

* this is the primary bead creating a linear 6-bead particle.

Data in Table 2 show that the translation of particles in glycerin can be treated as a viscous one ($Re < 1$). Computer code WinHYDRO++ gave generally higher values of the translational velocity than the measured ones, most probably due to neglecting wall effects in this theoretical method. The ratio of translational velocity of a linear 6-bead particle to the velocity of a single bead was equal in our study to 2.4. This value is close to the one presented by Zahn et al. (1994), which was about 2.2.

Relationships between translational velocities of the investigated particles and their fractal dimensions are given in Fig. 2. They can be approximated by linear correlations.

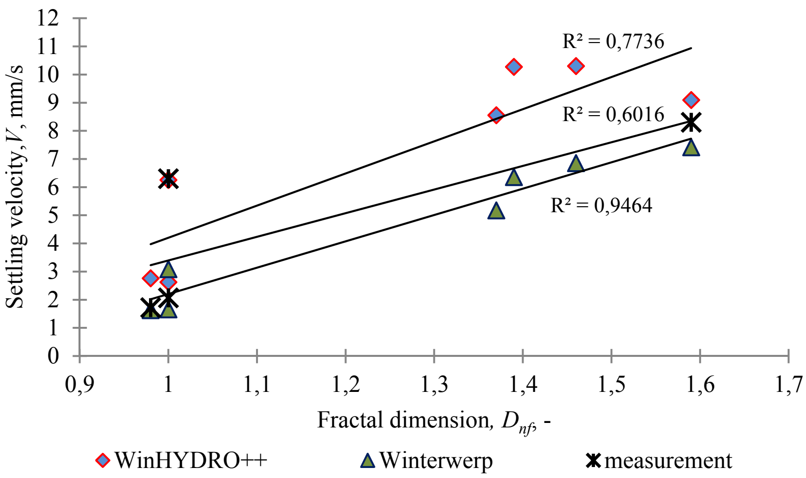

Fig. 2. Relationships between translational velocity of particles and their fractal dimensions

From Fig. 2 one may conclude that the higher the fractal dimension, the higher translational velocity. Values generated by the WinHYDRO++, as a function of fractal dimension, are higher than those calculated using Winterwerp's formula. In Fig. 3 deviations of the calculated velocities from the measured ones are depicted. It can be seen that their agreement is pretty good, excluding the result for a linear 6-bead particle, calculated using the Winterwerp's formula. The Winterwerp model seems to be equally good as the WinHYDRO++ model, except for the measured settling velocity equal to 6 mm/s for the linear 6-bead particle. The values calculated using the Winterwerp's formula gave underestimated results (on average by –22%) and those calculated using the WinHYDRO++ model overestimated the measured ones (on average by +24%).

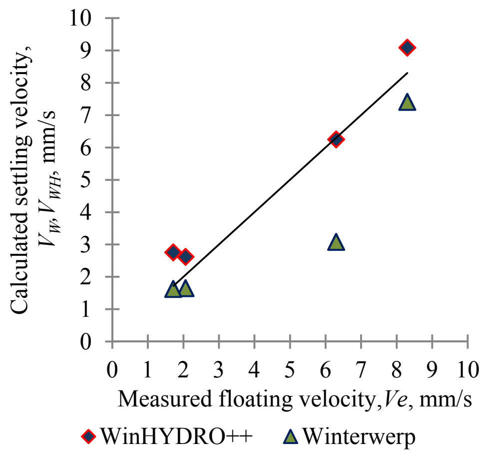

Fig. 3. Parity graph comparing calculated results with the experimental ones

In order to determine the impact of the configuration of the model bead particles (agglomerates) on translational velocity the WinHYDRO ++ was used. In Fig. 4-6, relationships between settling velocity, radius of gyration and fractal dimension are presented. Radius of gyration is defined here as:

$$R_g = \sqrt{\frac{\sum_{i=1}^{N} m_i \; r_i^2}{m}} \tag{4}$$

where m_i - mass of i-th primary particle, distant by r_i from the axis of gyration, m – total mass of all N particles in a given agglomerate.

In Fig. 4 one can observe linear correlations between variables as well as statistically significant coefficients of determination.

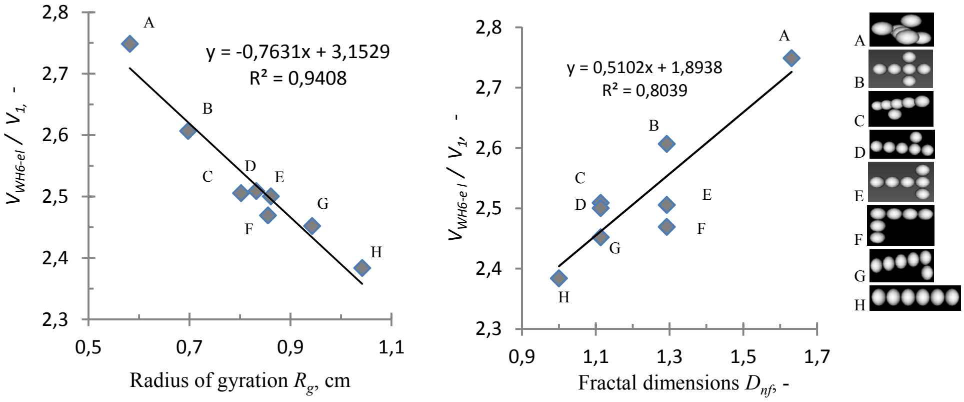

Fig. 4. Ratios of settling velocities $V_{WH\,6\text{-}el}$ (for aglomerate 6-bead particle) to the settling velocity V_1 (for 1-bead particle) calculated using WinHYDRO++ versus radii of gyration R_g and fractal dimensions D_{nf}

In Fig. 5 and 6, depicting the same relationships for more complex agglomerates, the linear correlation holds for the right hand side pictures only, showing the impact of fractal dimensions. The dependence of the settling velocity on agglomerate radius of gyration is better described by non-linear (quadratic) correlations.

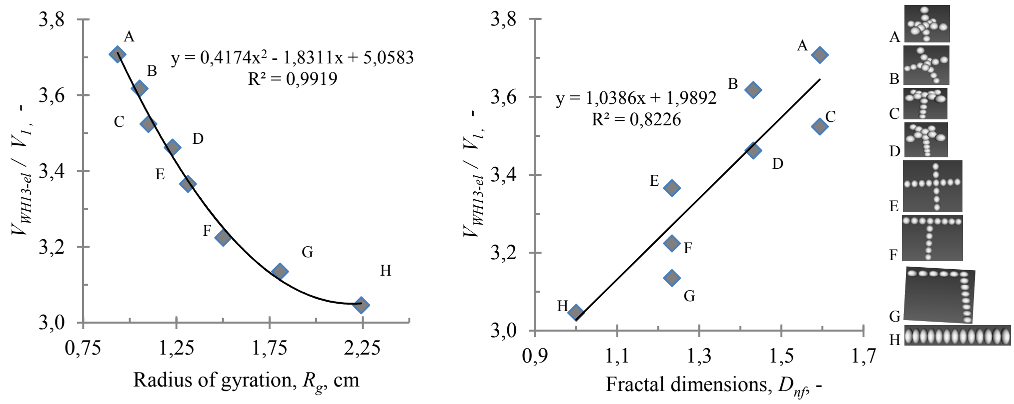

Fig. 5. Ratios of settling velocities $V_{WH\ 13\text{-}el}$ (for aglomerate 13-bead particle) to the settling velocity V_1 (for 1-bead particle) calculated using WinHYDRO++ versus radii of gyration R_g and fractal dimensions D_{nf}

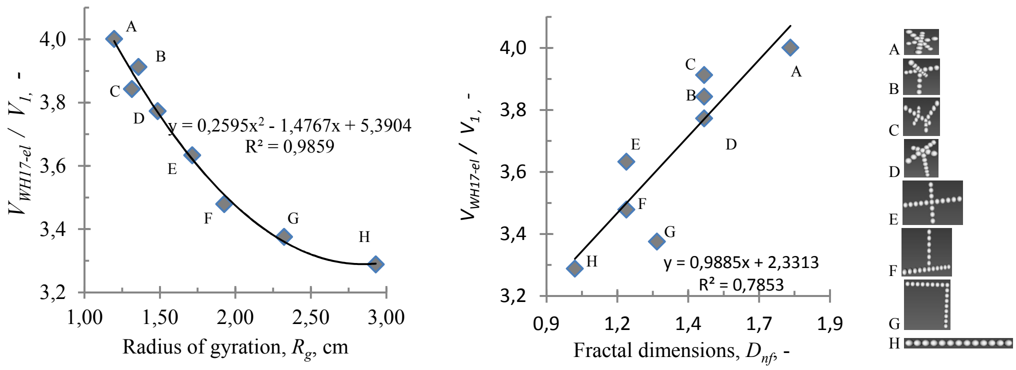

Fig. 6. Ratios of settling velocities $V_{WH\ 17\text{-}el}$ (for aglomerate 17-bead particle) to the settling velocity V_1 (for 1-bead particle) calculated using WinHYDRO++ versus radii of gyration R_g and fractal dimensions D_{nf}

4. CONCLUSIONS

The Winterwerp's formula for settling velocity of cohesive flocs is generally consistent with our experimental data but it cannot be applied for agglomerates with fractal dimensions less than 1.4, i.e. quasi-linear agglomerates.

Computer code WinHYDRO++ can be applied for calculation of settling (floating) velocities at low Reynolds numbers, but it neglects an agglomerate position (relative to the direction of translation) at calculations of translational velocities.

Agglomerates constructed from primary bead particles settle or float, at low Reynolds numbers, depending on their arrangement and fractal dimensions as well as the other parameters taken into account in the Stokes formula. Radius of gyration seems to be a better parameter to describe the settling velocities than the fractal dimension.

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